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RELATIVISTIC DESCRIPTION OF FEW BODY SYSTEMS

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RELATIVISTIC DESCRIPTION OF FEW BODY SYSTEMS

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In this talk I will discuss how relativistic meson theory is developed and applied to the electromagnetic description of the two nucleon system. These techniques are being extended to the three body system and very similar methods have been applied to nuclear matter, but I will not review this work here.

It is no longer possible to regard relativistic meson theory as fundamental. I view it as a consistent relativistic theory of effective interactions between selected quark clusters, which are treated structureless particles. The emphasis is on the words "consistent" and "relativistic." This means that I will insist that the theory be manifestly covariant at every step (which means that I will not discuss approaches based on time-ordered perturbation theory), and that the electromagnetic current operator J^μ and the relativistic "potential" or kernel V be consistent with one another. Some attempt is made to allow for the structure of the mesons and nucleons by inserting phenomenological form factors at the vertices and, in some cases, using simple phenomenological functions for self energies (which, strictly speaking, spoils the consistency), but the basic equations of the theory are obtained from a Lagrangian for point-like mesons and nucleons. The justification for using such a theory today is that it gives a calculable theory of nuclei which employs the degrees of freedom most apparent in nuclear physics, and which through detailed comparison with experiment can help us uncover those phenomena which require the explicit use of quark degrees of freedom.

I will begin with a discussion of how relativistic equations can be developed from a consideration of the summation of infinite classes of diagrams. Section 2 will summarize some applications of relativistic few body equations, including a brief account of some recent fits to the nucleon-nucleon phase shifts not yet published. Then, in Section 3, I will review some applications involving electromagnetic interactions.

1. RELATIVISTIC WAVE EQUATIONS

1.1 Types of Equations

Relativistic equations can be written in the following very general form

$$M = V + VGM \quad (1)$$

where M is the scattering amplitude, V is the kernel or relativistic

potential, and G the propagator. If V is in some sense small, Eq. (1) can be solved by iteration as shown diagrammatically in Fig. 1 for two particles. The equation can be regarded as a means of summing a generalized Born series, or summing an infinite number of diagrams. If V is small, the solution to (1) will not differ significantly from taking V alone. However, when V is large, the Born series will not exist, but the solution to (1) will. In this sense relativistic equations enable us to treat non-perturbative problems.

$$M = V + V M$$

$$= V + V V + V V V + \dots$$

FIGURE 1

Bound state wave functions can be obtained from the residues of the bound state poles of M . Near the bound state pole at M_B ,

$$M(p, p', W) = \frac{\Gamma(p)\Gamma^*(p')}{M_B^2 - W^2} + R \quad (2)$$

where p and p' are the relative 4 momenta of the final and initial state respectively, W is the total CM energy, and R is a remainder function regular at M_B^2 . Substituting (2) into (1) one can obtain both the bound state wave equation and the normalization condition

$$\Gamma = V G \Gamma \quad (3)$$

$$1 = \int \Gamma^* \left(\frac{dG}{dW^2} - G \frac{dV}{dW^2} G \right) \Gamma \quad (4)$$

The relativistic wave function ψ is related to the vertex function Γ by

$$\begin{aligned} \psi &= G \Gamma \\ \Gamma &= V \psi \end{aligned} \quad (5)$$

To find the relativistic kernel V from an infinite class of diagrams, one must first decide on what class of diagrams to sum, and then introduce a scheme for organizing the sum. I will assume that the smallest class of diagrams which will describe the dynamics

adequately is the sum of all ladder and crossed ladder Feynman diagrams (with form factors at the vertices and on the propagators). In particular, it is known that crossed ladders make important contributions, and therefore the ladder sum alone is certainly not adequate. If particle production and inelasticities are to be treated explicitly, a larger class of Feynman diagrams including self energy contributions is almost certainly necessary, but for elastic processes the ladder and crossed ladder sum may be sufficient. This sum, up to 6th order in the coupling constant, is shown in Fig. 2 for the case of two heavy nucleons exchanging a light meson. The ladder diagrams are (a), (b), and (d); all others are crossed ladders.

The way in which this sum is organized now depends on how the two body propagator G is defined. In the most general case, the propagator G is constrained according to some covariant prescription so that it

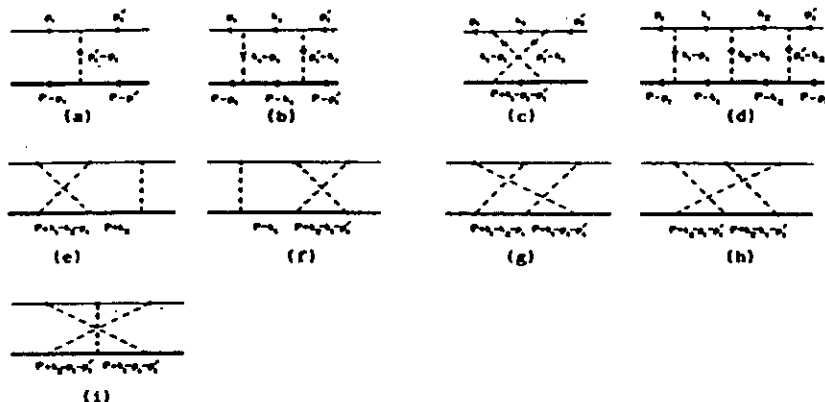


FIGURE 2

depends on only the relative 3 momentum instead of the relative 4 momentum. The advantage of such an approach is that the number of free variables is thereby reduced, making the resulting integral equation simpler to solve and easier to interpret. The kernel V corresponding to the constrained G is then the sum of all diagrams which cannot be obtained by iterating lower order kernels as shown in Fig. 1 (where the constrained propagator is represented by a vertical dotted line cutting the two nucleons). Hence the precise definition of V depends on the definition of G . The kernel up to 4th order is shown diagrammatically in Fig. 3. The first diagram (3a) is the one boson exchange (OBE) contribution, the second (3b) is the difference between the full box diagram and the first iteration of the OBE, which is called the subtracted box, and the third (3c) is the crossed box. If the uncon-

strained 2 body propagator is used, as in the Bethe Salpeter (BS) equation²⁾, then the full box is obtained after one iteration of the OBE,

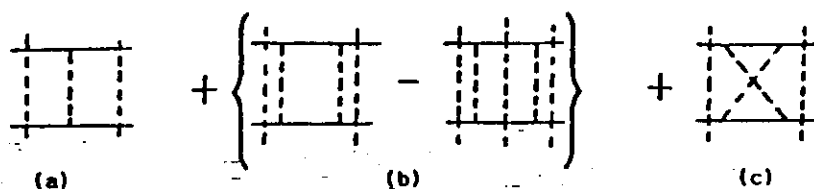


FIGURE 3

so the subtracted box is zero. With constrained propagators, the full box is not obtained after one iteration, so the subtracted box must be added. In 6th order subtracted boxes and subtracted crossed boxes coming from Figs. (2d-f) must be included in the kernel as well as the fully crossed ladder diagrams (2g-i), and so on to all orders. What the relativistic equation has done for us is to replace the full sum in Fig. 2, which certainly does not converge for large coupling constants, with a sum like that shown in Fig. 3 for the kernel. The procedure will only work if the sum for the kernel converges rapidly. Before I discuss this important issue, I will review a number of popular choices for the propagator G .

Four choices of G are summarized in Table I. The BS equation conserves 4 momentum in the intermediate state, so it remains on the energy shell defined by $P_0 = W$, where W is the initial energy of the two body system and P_0 is the total energy in the intermediate state (both in the CM system). This leaves all four components of the relative 4-momentum, $p = \frac{1}{2}(p_1 - p_2)$, unconstrained. Alternatively, if we restrict one particle²⁾ to its positive energy mass shell (say particle 2)³⁾, then $P_0 = W$ and $p_{20} = (M^2 + \vec{p}^2)^{1/2} = E_p$ fixes the relative energy in a covariant way

$$p_0 = \frac{1}{2}W - E_p$$

leaving only the three components of \vec{p} as free variables. If we wish to restrict both particles to their mass shells, we must drop the requirement that $P_0 = W$, or go off the energy shell. One way of doing this was developed by Logunov and Tavkhelidze⁴⁾ and by Blankenbecler and Sugar⁵⁾; a variation of this method is due to Todorov⁶⁾. An advantage of this approach is that the number of spin degrees of freedom is reduced because both particles are on shell and hence have only two spin degrees of freedom. Finally, the light front equation⁷⁾ comes from a different approach in which field theory is quantized on the light front⁸⁾, which loosely speaking refers to quantizing fields at equal values of $\tau = t+x$ (the velocity of light c is taken equal to unity and x can be any one of the three directions in space). The variable conjugate to τ is $p_- = E - p_x$ which now plays a role similar to that usually played by the energy,⁹⁾ so that this approach bears a

close formal resemblance to old fashioned time ordered perturbation theory, where all particles are on the mass shell, but intermediate

Table I
Relativistic Two Body Equations

Name	Description of G	Number of Variables	
		Momentum	Spin
Bethe-Salpeter (BS)	On energy shell Both particles off mass shell	4	$4 \times 4 = 16$
Particle 1 off shell (G_1)	On energy shell One particle off mass shell	3	$2 \times 4 = 8$
Blankenbecler-Sugar Logunov-Tavkhelidze (BSLT)	Off energy shell Both particles on mass shell	3	$2 \times 2 = 4$
Light Front (LF)	Off $p_- = E - p_x$ shell Both particles on mass shell	3	$2 \times 2 = 4$

states are off the energy (p_- in this case) shell. However, while there is a formal resemblance between τ ordered diagrams and time ordered diagrams there is a profound difference which cannot be over-emphasized. The τ ordered formalism is manifestly covariant at every step while time ordered perturbation theory breaks covariance. This is related to the fact that τ is invariant under boosts in the x direction, while t is not. A disadvantage of the light front formulation is that it breaks manifest rotational invariance. Several authors have used LF techniques in recent years.⁹⁻¹²⁾

The extension of relativistic equations to more than two bodies is a subject of increasing importance. All of the equations mentioned above can be extended but the BS equation has 4 (N-1) integration variables while the constrained equations (in common with non-relativistic equations) have only 3 (N-1). It is important that any n body system of $n < N$ particles must be dynamically independent of the others when the others are beyond the range of forces. A serious deficiency of the BSLT equation is that it does not satisfy this requirement.

slowski and Weber¹³⁾ have shown that the three body LF equation satisfies the cluster property, and it has recently been shown¹⁴⁾ the three body generalization of the G_1 equation also has this property.

I wish to emphasize that the constrained equations should not be regarded as an approximation to the BS equation. From a relativistic point of view, all of the equations are equally good starting points; the question of which equation is "best" will depend on other criteria, such as how rapidly the series for the kernel converges.

1.2 Convergence of the infinite series for V

I now want to discuss the convergence of the infinite series for the first three terms of which are shown diagrammatically in Fig. 3. The terms in the $2n^{\text{th}}$ order kernel cancel among themselves, making the full $2n^{\text{th}}$ order kernel smaller than a typical $2n^{\text{th}}$ order term, conclude that the series for V converges more rapidly than if no cancellations were present.

As an example of how these ideas work, consider the case of a light particle m interacting with a very heavy, neutral scalar particle M . It has been known for many years¹⁵⁾ that in the limit as $M \rightarrow \infty$ the terms and crossed ladders cancel in such a way that the total result can be obtained by iterating the OBE kernel with the heavy particle restricted to its mass shell (other constrained prescriptions also work; $M \rightarrow \infty$ they are equivalent to putting the heavy particle on shell). This means that the irreducible kernel reduces exactly in this limit

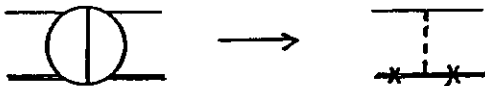


FIGURE 4

as shown in Fig. 4 (there the x means the particle is on-shell). The terms of the diagrams shown in Fig. 3, it means that the subtracted terms and the crossed box exactly cancel when $M \rightarrow \infty$. Furthermore, the cancellation takes place in every order, leaving the OBE to give the exact relativistic one body equation for the light particle m as if m has spin $\frac{1}{2}$, Klein Gordon if m has spin zero moving in an instantaneous potential.

Unfortunately, the BS equation does not have a one body limit in this sense. In the BS equation, the subtracted box is exactly zero,

leaving nothing to cancel the contributions from the crossed box. This happens in every order, so that the BS kernel in the $M \rightarrow \infty$ limit remains an infinite sum.

When both particles have spin, or when the heavy one is charged, such general results have not been proved, but may very well be true in some cases. For example, the cancellation has also been observed to work in 4th order for two spin $\frac{1}{2}$ particles in QED.^{3,16-17} The cancellation also occurs in 4th order for two heavy spin $\frac{1}{2}$ nucleons exchanging pseudoscalar, isovector pions, provided the π -N interaction is treated in a manner consistent with chiral symmetry.¹⁵

While a study of the rate of convergence of the infinite series for V suggests that the Bethe Salpeter equation is not optimal, the results do not clearly distinguish between the three-dimensional equations described in Table I. The choice of propagator depends on other considerations as much as it does on questions of convergence. My own preference for the choice which puts all particles on shell but one is illustrated in Figures 5 and 6. The dominant contribution to the triangle diagram which contributes to the deuteron form factor is obtained

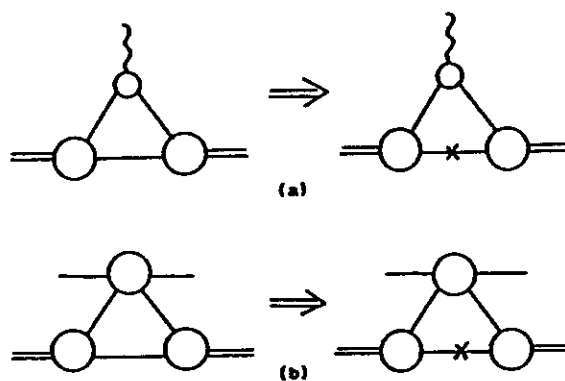


FIGURE 5

if one restricts the spectator to its mass shell, as shown in Fig. 5a. This is the relativistic impulse approximation (RIA). A similar consideration shows that the dominant contribution to the single scatter-

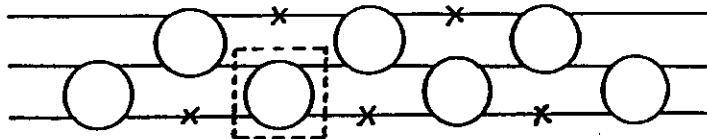


FIGURE 6

diagram in p d scattering also comes from the process in which the spectator is on shell (Fig. 5b). Examination of three body scattering, viewed as a succession of two body interactions, leads to the same conclusions. Figure 6 shows how the three body amplitude is driven by a two body amplitude in which only one nucleon in the initial state and one in the final state is off-shell, and since the spectator is always on-shell, it is obvious why this three body equation satisfies the cluster property.

The above discussion may suggest that the prescription of placing the spectator on-shell is an approximation to a more exact procedure in which it is allowed to be virtual. As in the case of the BS equation, I believe that this is not the case, and that diagrams in which the spectator is on-shell should be viewed as the first term in a well defined series of approximations which are expected to give the exact result when summed to all orders.

Unfortunately, no approach to the relativistic problem seems to be without disadvantages. If one nucleon is restricted to its mass-shell, we wish to retain the Pauli principle for identical nucleons, then must antisymmetrize either the interaction or the propagator explicitly. This means that, in the one boson exchange approximation, we must include both of the diagrams shown in Fig. 7. While this is



FIGURE 7

straightforward in principle, it introduces complications in practice. The use of the second diagram, Fig. 7b, has unphysical singularities. It can be shown that these singularities cancel when higher order contributions are included, so one possible approach is to treat these singularities as a principal value. Alternatively, one may redefine the propagator in such a way that the singularities are absent, and this is the approach taken in the work described below. Another disadvantage of the approach in which one particle is placed on shell is that the two-body equation has spurious states for the case when the covariant mass of the two body system is zero; while this feature is undesirable in principle, it introduces no practical problems.

2. APPLICATIONS OF RELATIVISTIC FEW BODY EQUATIONS

I will only discuss very briefly a few applications of these equations to calculations of the bound state and scattering properties of

few body systems.

2.1 The Two Nucleon System

Fits to the two nucleon phase shifts for energies below 300 MeV have been obtained by Fleischer and Tjon¹⁸⁾ and by Zuilhof and Tjon¹⁹⁾ using the BS equation in OBE approximation. These fits have been extended to energies up to 1000 MeV by van Faassen and Tjon²⁰⁾, who describe the inelasticity by including NA intermediate states. Fits to the phase shifts below 400 MeV have also been obtained by Gross and Holinde using the G_1 equation²¹⁾, and I want to describe these new, unpublished results in a little more detail.

The relativistic kernel employed in Ref. (21) consists of an OBE model with only four mesons: π , σ , ρ , and ω . (Instead of varying the σ mass and coupling constant, two sigmas of fixed masses at 350 MeV and 760 MeV were chosen, and the couplings of each varied.) Form factors were used at the meson-nucleon vertices, and a form factor was also used with the off-shell nucleon propagator to improve convergence. While only four mesons are used, the number of parameters varied is similar to that used in conventional OBE models with more mesons, because two mixing parameters were used which do not appear in usual approaches. These are λ and μ , where λ varies the fraction of γ^5 to $\gamma^5\gamma^\mu$ coupling at the πNN vertex¹⁾, and μ varies the fraction of $\sigma^{\mu\nu}$ and P^μ couplings at the ρNN vertex. The πNN and ρNN couplings were defined so that when the nucleons are on their mass shell the coupling is strictly independent of the value of the mixing parameter (for the ρNN coupling one uses the Gordon decomposition, which only holds on shell, to transform $\sigma^{\mu\nu}$ into P^μ). Hence, dependence of the results on these two parameters is a direct measure of the possible importance of off shell effects, and we find that such effects are large. In fact it is because of the splitting between the 1S_0 and 3S_0 phase shifts introduced by the λ dependence that we do not need the isovector-scalar meson δ in these fits.

Another novel feature of the G_1 equation employed in these fits is that the off-shell Dirac nucleon has four spin states; two for its positive energy state and two for its negative energy state. One can separate the positive and negative energy "channels", giving a coupled set of equations of the following form¹⁾

$$\begin{aligned}(2E_k - W) \psi^+ &= V^{++} \psi^+ + V^{+-} \psi^- \\ W \psi^- &= V^{-+} \psi^+ + V^{--} \psi^-\end{aligned}\tag{6}$$

In Ref. (21) the approximation $V^{--} = 0$ was taken, yielding the "solution"

$$(2E_k - W) \psi^+ = (V^{++} + \frac{|V^{+-}|^2}{W}) \psi^+\tag{7}$$

which shows how the negative energy channel, which makes no contribution to the asymptotic states, modifies the effective interaction at short range. To obtain Eq. (7) one uses the fact that the matrix potential in Eq. (6) is hermitian.

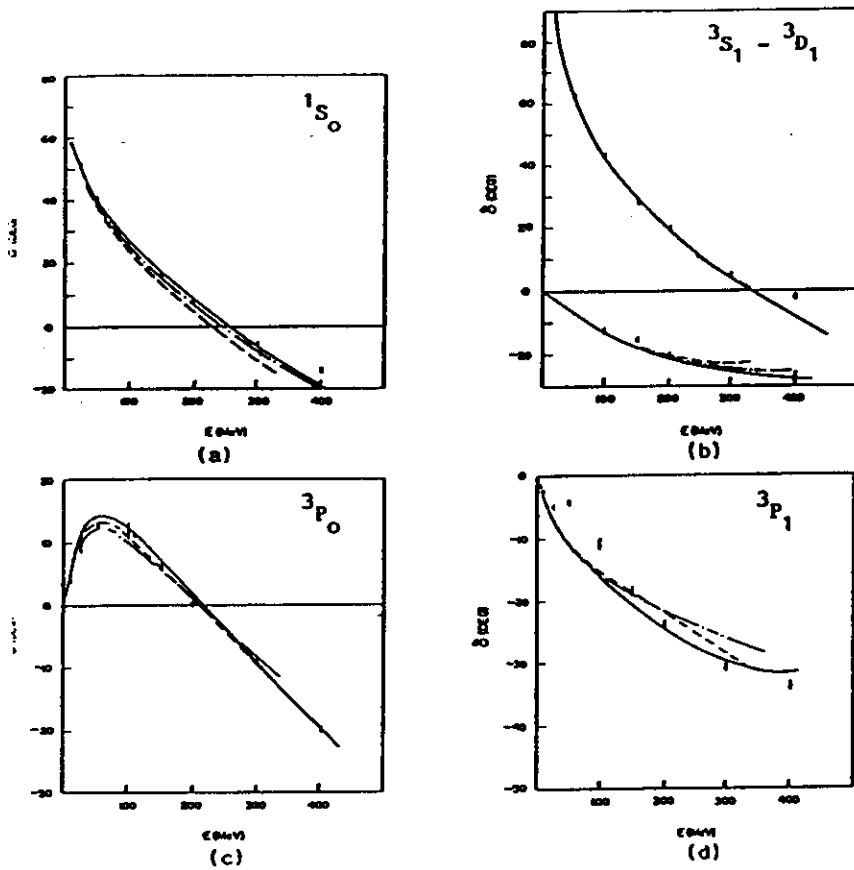


FIGURE 8

Another novel feature of this treatment is the presence of virtual "wrong" symmetry channels. These channels, which are symmetric under the interchange of three momentum and spin indices, are not forbidden by the Pauli principle in a region of phase space where the relative energy p_0 is not zero, but must vanish when $p_0 = 0$ (i.e. when both particles are on shell). It turns out that it is necessary to explicitly antisymmetrize the potential to guarantee that these channels are really virtual (i.e. are zero when $p_0 = 0$), and this has been done in Ref. (21) as discussed above. These effects are present only for partial waves where $L=J$.

The result of all these considerations is that the coupled equations (6) contain four channels for all partial waves. For partial waves which are coupled by the tensor force in non-relativistic theory (e.g. $^3S_1 - ^3D_1$) there is a doubling due to the presence of negative energy states. For partial waves which were formally uncoupled, there is both a doubling due to the presence of negative energy states, and due to coupling to virtual wrong symmetry channels.

I now turn to a brief description of the results of Ref. (21). Figure 8 shows the fits to the 1S_0 , $^3S_1-^3D_1$, 3P_0 , and 3P_1 phase shifts below 400 MeV. The solid lines

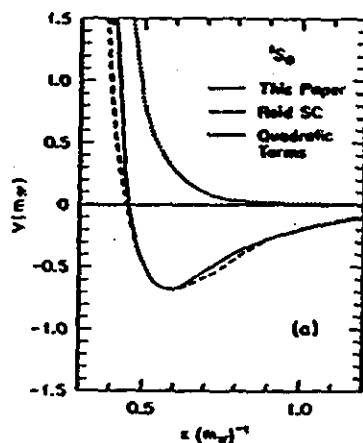


FIGURE 9

pulsion at short range¹⁾

Figure 9, taken from Ref. (1), shows that

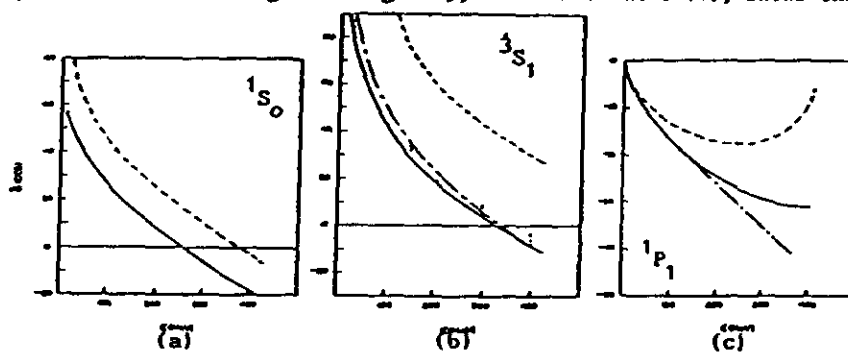


FIGURE 10

By varying the parameters in the relativistic kernels in Ref. (21), or by varying the size of the V^{+-} potentials, relativistic and off-mass-shell effects can be investigated. It has been known for over ten years that a major effect of the negative energy channel is to provide re-

quadratic terms, which are the squared terms in the effective potential given in Eq. (7), are large and repulsive. As expected, we see the same effect in the actual fits to the phase shifts, as shown in Fig. 10. In this figure the solid lines are the original fits, the dashed lines are the results when $V_{\pi}^{+-} = 0$, and the dashed/dotted lines are the results when only $V_{\pi}^{+-} = 0$. In the 1S_0 channel there is no noticeable difference between these two cases. Note the strong isospin dependence of the quadratic repulsion, which is seen by comparing the effect in the 1S_0 and 3S_0 channels. The figure also shows the V_{π}^{+-} terms coming from mesons other than the pion are, in some cases, of comparable importance to the large V_{π}^{+-} contribution. One effect which is probably due to this repulsion is that the ω coupling constant which emerges from this fit is

$$\frac{g_{\omega}^2}{4\pi} = 9.52$$

is similar to that obtained in Ref. (19), and considerably smaller than that needed in many non-relativistic OBE fits.

The effect of the coupling to virtual wrong symmetry channels for 1P_1 and 3P_1 partial waves is shown in Fig. 11, where the dotted

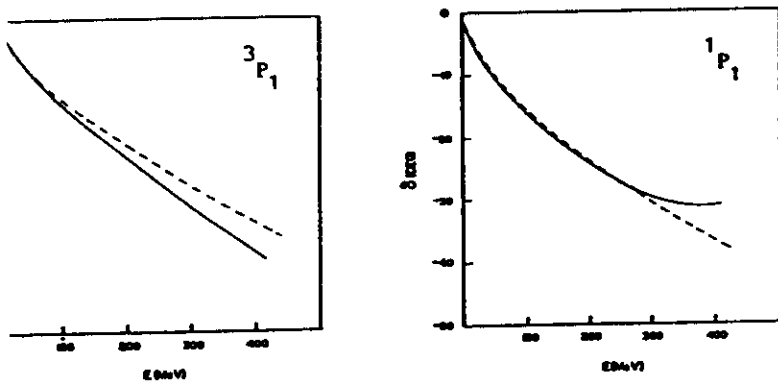


FIGURE 11

shows the effect without the coupling. In these cases the phase shifts at 300-400 MeV are well outside of the error bars for the experimental data; in the 1P_1 case this coupling alters the shape of the phase shifts in a helpful way and in the 3D_1 case (not shown) it gives helpful repulsion.

What can one conclude from all this? While it is somewhat hard to say, it is my view that almost any equation with a sufficient number of bosons and about 10 parameters can be made to fit the NN phase shifts below 400 MeV. This does not mean that the differences between relativistic equations are small, or that the relativistic

effects themselves are small. In fact, such differences are known to be numerically large^{19,24)}. Rather, it appears that adjustments of 10 parameters can largely compensate for these differences. Since the parameters have physical significance, the extent to which their adjusted values agree with values determined from other physical processes could be a test of the validity of the equation. Perhaps a better method is to see how well a given equation, "tuned" to the two body problem, is able to describe the three body problem, nuclear matter, and other calculable systems.

2.2 Other Systems

There is evidence that relativistic effects increase the binding of the three body system, reducing the discrepancy between the calculations and observed binding. Coester and Wiringa²⁵⁾ found an increase of 1.7 MeV for the triton binding and 4.3 MeV for the alpha binding, and Rupp²⁶⁾, using a separable BS equation, found similar effects. Unfortunately, neither calculation can be regarded as treating the dynamics in a realistic way. A fully relativistic treatment of the three body system, with realistic dynamics consistent with the two body problem, is needed. Such a calculation, using the three body version of the G_1 equation is possible.¹⁴⁾ This approach satisfies the cluster property, yields relativistic Faddeev equations with the same number of momentum variables as the non-relativistic equation, and (as in the non-relativistic case) can be reduced analytically to a two dimensional integral equation for coupled partial waves²⁷⁾.

Relativistic calculations of nuclear matter^{28,29)}, and the NN system³⁰⁾ have also been carried out. These show interesting effects due to relativity which I will not discuss here.

3. APPLICATIONS INVOLVING ELECTROMAGNETIC INTERACTIONS

I now turn to the treatment of electromagnetic interactions of few body nuclei using amplitudes obtained from the equations discussed above. Before I show some results, I want to emphasize that the relativistic impulse approximation (RIA) shown in Fig. 5 above and Fig. 12 can be further decomposed into time ordered pieces which include the usual impulse approximation, Fig. 12, plus two zigzag diagrams often referred to as pair contributions, Fig. 12b.

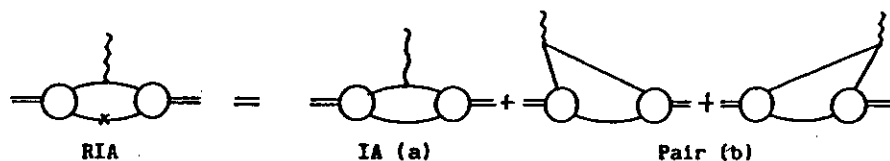
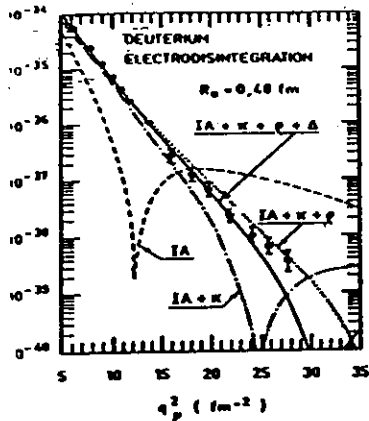


FIGURE 12

must be exercised in comparing relativistic calculations with non-relativistic ones; in the former the pair terms are included in the RIA; in the latter they are added to other diagrams and considered to be exchange contributions³¹⁾.

The classic example of the importance of MEC is the radiative neutron capture cross section (the time reversed threshold photodisintegration process³²⁾ and the electrodisintegration of the deuteron to an np final state very near threshold³³⁾.



Recent data on the latter process is shown in Fig. 13, from Ref. 34. The curve labeled IA is the non-relativistic impulse approximation and has a minimum at $Q^2 \approx 12 \text{ fm}^{-2}$ due to destructive interference between the deuteron S and D state contributions. The other curves show the effect of MEC, which are dominant here. However, in both this case and radiative neutron capture, the pair terms are the dominant contribution to the non-relativistic MEC, and in this sense these processes are also evidence for the importance of relativistic corrections. The argument is not conclusive, however, because the pair terms can be reduced by employing a $\gamma^5 \gamma^\mu$ coupling for the pion, which trans-

FIGURE 13

the pair terms into a $\gamma\pi\pi\pi$ contact term, which must be regarded MEC.

relativistic corrections have been most extensively studied in π -deuteron scattering. Corrections to the magnetic moment, dipole moment, and deuteron charge radius have been calculated^{35,36)} and it was found many years ago that the corrections at low momentum transfer are helpful in bringing the measured slope of the pion charge form factor at $Q^2 = 0$ into line with electron-deuteron scattering data.³⁷⁾

The behavior of the form factors at high momentum transfer Q^2 has recently been studied using the BS formalism³⁸⁾, and the G_1 formalism³⁹⁾. The principal results of these two calculations are shown in Fig. 14 (from Ref. 38) and Fig. 15 (from Ref. 36), which both show the ratio of the relativistic calculation of the electric A structure function to the non-relativistic calculation for identical wave functions. Note that both the horizontal and vertical scale are quite different; the dotted boxes shown in each figure cover the same region. This shows that these two calculations are in rough agreement, and show that relativistic effects make the form factors smaller at high Q^2 than

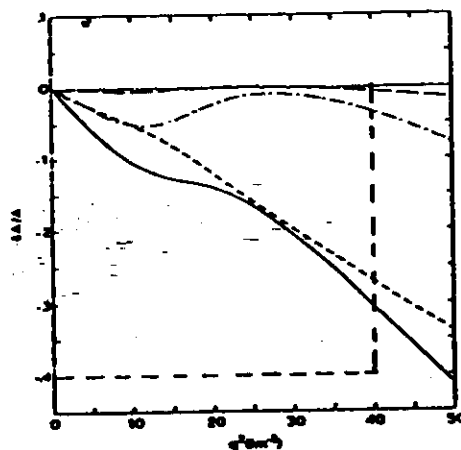


FIGURE 14

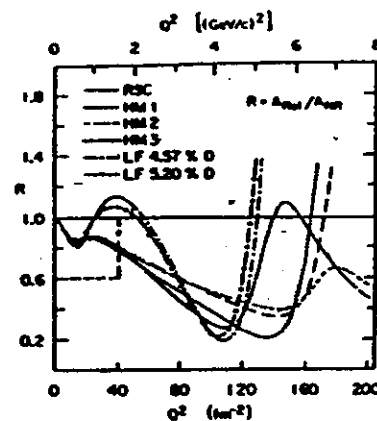


FIGURE 15

non-relativistic calculations, further widening the discrepancy between theory and experiment. However, results from the LF formalism by Frankfurt and Strickman¹²⁾ show the opposite effect. Grach and Kondratyuk³⁹⁾ also use the LF formalism, and are able to produce effects similar to Ref. (38,36) or (12) depending on which nucleon form factors they use. Still another approach has been taken by Troitski and Trubnikov⁴⁰⁾.

Recent measurements of the magnetic structure function, B , from Saclay out to momentum transfers of $1 (\text{GeV}/c)^2$ show the same trends; the RIA calculations fall considerably below the data⁴¹⁾.

It is clear that measurements of the neutron charge form factor, G_{En} , and better measurements of G_{Ep} at high Q^2 are essential before the data on the deuteron form factor can be fully exploited. If the discrepancies remain, then we have evidence for large $I = 0$ meson exchange currents (which could be due to the $\rho\omega\gamma$ interaction, or to two meson exchange terms) or for 6 quark components in the deuteron wave function.

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